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NUCLEAR POLARIZATION AND THE EQUIVALENCE PRINCIPLE*

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In this paper, we analyze the nuclear polarization of the spin-polarized Dy_6Fe_{23} used in our two equivalence principle (EP) experiments. From this we infer the equivalence of polarized Dy in the earth's gravitational field to be good to 10^{-3} and in the solar field to be good to 1.4×10^{-2}. To increase the nuclear polarization in order to have better EP tests, we propose to use a dilution refrigerator to lower the temperature to 10 mK. We present a thorough analysis of our experimental scheme together with a discussion of perspectives.

1. Introduction

The equivalence principle is a fundamental assumption of the general relativity and metric theories of gravity. It is the cornerstone of universal gravitation. The precision of its experimental validity puts an important constraint on gravitation theories and particle theories. Possible deviations from equivalence would give clues to the microscopic origin of gravity or to some new fundamental force(s). Polarized experiments for long-range force measurements play an especially important role in this respect.[1]

In the new general relativity of Hayashi and Shirafuji,[2] the coupling with an antisymmetric field leads to a universal spin-spin interaction. From gauging a subgroup of the Lorentz group, Naik and Pradhan[3] proposed a similar interaction. Around 1980, the particle physics community began to realize the possible existence of Goldstone bosons and/or pseudo-Goldstone bosons.[4–7] These bosons generate (semi-)long-range forces of monopole-monopole type, monopole-dipole(spin) type, and dipole-dipole (spin-spin) type. The recent issue of the fifth force arises from speculation about the existence of semi-long-range coupling to baryon number/hypercharge/lepton number.[8] Attempts have been made to construct models of long-range forces in higher dimensional Kaluza-Klein type theories[9] and superstring theories.[10] More recently, an attempt to generalize the Nambu-Goldstone mechanism shows that the restoration of a spontaneous violation of a "fact", such as the "fact" that translation generators in different directions commute, implies the existence of

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a massless excitation, and therefore, a long range force.[11] All the above cases can be explored experimentally by gravitation-type experiments on macroscopic bodies — Eötvös-type experiments, Galileo-type ("free-fall") experiments and Cavendish-type experiments.

To investigate the equivalence principle for spin-polarized bodies or to probe the mass-spin (monopole-spin/baryon-number-to-spin) interactions, we have used both a beam balance[12] and a torsion balance[13] to test a magnetically shielded spin-polarized body of Dy_6Fe_{23}. From these results, we have inferred that, to an accuracy of 5×10^{-3}, the polarized electron falls at the same rate as unpolarized bodies in the earth's gravitational field, and that it falls at the same rate as unpolarized bodies in the solar gravitational field with a deviation from this unity ratio estimated at $(3 \pm 4) \times 10^{-2}$.

While most of the polarized spins of the Dy_6Fe_{23} are due to electrons, there are some due to nuclear polarization. In this paper we estimate the extent of the nuclear polarization of Dy_6Fe_{23}, analyze the implication of equivalence principle tests with Dy_6Fe_{23} on nuclear spins, and present our proposal and progress on improving these tests.

In Sec. 2, we calculate the value of the nuclear polarization of Dy_6Fe_{23} and $HoFe_3$, and discuss equivalence principle tests on nuclear spins. In Sec. 3, we propose to use a dilution refrigerator to obtain high nuclear polarization and present our experimental scheme together with our present progress. In Sec. 4, we conclude with remarks and perspectives.

2. Nuclear Polarization of Dy_6Fe_{23} and $HoFe_3$

Dy_6Fe_{23} is ferrimagnetic. The magnetic field at the Dy nuclei is approximately that of the free Dy ions, while the field at the Fe nucleus is typically very small. For Dy, we can assume a field of 5.5 MG and for Fe, 200 kG. For dysprosium, there are two naturally occurring isotopes that have nuclear magnetic moments: ^{161}Dy with $I = 5/2$, $\mu_I = -0.46\,\mu_N$ and 18.88% abundance; ^{163}Dy with $I = 5/2$, $\mu_I = 0.64\,\mu_N$ and 24.97% abundance.[14] For iron, there is only one naturally occurring isotope that has a nuclear magnetic moment, i.e., ^{57}Fe with $I = 1/2$, $\mu_I = 0.09\,\mu_N$ and 2.19% abundance.

At room temperature the iron nuclear polarization is negligible compared to dysprosium polarization. The dysprosium has a fractional polarization of

$$P = \frac{-e^{-\mu_I H/kT} - \frac{3}{5}e^{-\frac{3}{5}\mu_I H/kT} - \frac{1}{5}e^{-\frac{1}{5}\mu_I H/kT} + \frac{1}{5}e^{\frac{1}{5}\mu_I H/kT} + \frac{3}{5}e^{\frac{3}{5}\mu_I H/kT} + e^{\mu_I H/kT}}{e^{-\mu_I H/kT} + e^{-\frac{3}{5}\mu_I H/kT} + e^{-\frac{1}{5}\mu_I H/kT} + e^{\frac{1}{5}\mu_I H/kT} + e^{\frac{3}{5}\mu_I H/kT} + e^{\mu_I H/kT}} \tag{2.1}$$

when the electrons are polarized. Since at room temperature $\mu_I H/kT = 3.08 \times 10^{-4}$

for ^{161}Dy, the polarization can be approximated up to the linear term in $\mu_I H/kT$ to be

$$p = \frac{7}{15}\mu_I H = 1.6 \times 10^{-4}. \tag{2.2}$$

For our polarized sample of Dy_6Fe_{23}, the polarization of polarizable electrons of Dy is 0.4. Hence 0.64×10^{-4} of ^{161}Dy nuclei are polarized. Counting all isotopes, 0.34×10^{-4} of Dy nuclei are polarized. Hence the polarized mass of Dy compared to the polarized mass of electron is

$$\frac{0.34 \times 10^{-4} \times 1860 \times 162}{5 \times 0.4} = 5.1. \tag{2.3}$$

Therefore the equivalence of polarized Dy would be 5 times better than polarized electrons if there were no spurious cancellations. From our experimental result in Refs. 12 and 13, the equivalence of polarized Dy in the earth's gravitational field is good to 1×10^{-3} and in the solar field to be 1.4×10^{-2}. Table 1 compiles these results together with results of previous experiments on polarized nuclei. To avoid possible spurious cancellations, we need to perform experiments at two different temperatures.

The holmium ion has a larger magnetic field at nucleus than that of the dysprosium ion, and ^{165}Ho is 100% abundant with a large nuclear moment: 4.173 μ_N. We currently use a holmium iron compound in our equivalence principle test, in order to enhance nuclear polarization. At room temperature an electron-polarized holmium atom has nuclear polarization $p = 3/7\ (\mu_I H)/(kT) = 1.70 \times 10^{-3}$, a polarization about 20 times larger than that of dysposium. Hence we are expecting to have a result 20 times more accurate for polarized Ho nuclei in the near future.

To obtain even more polarization, we need a low temperature. In a previous paper,[18] we presented a scheme of doing the equivalence principle experiment using a dilution refrigerator. In the next section, we present a thorough analysis of various specific points.

3. Millikelvin Equivalence Principle Experiments

The investigation of the spin-dependent forces involving baryons requires bodies in which the nuclei are polarized. Large nuclear polarizations can be achieved either by static means, where the body is cooled to a sufficiently low temperature that the Boltzmann distribution in the hyperfine levels corresponds to a large nuclear polarization, or by dynamic means, where energy sources such as microwaves disturb the populations of the hyperfine levels from their thermal equilibrium values. For a first experiment the static method seems most suitable since the absence of microwave and other fields means there will be fewer sources of systematic error, fewer stability problems, and less confusion regarding the results.

Table 1. A compilation of the equivalence principle experiment on polarized nuclei.

Polarized Nucleus	Experiment	Precision	Reference
Cs	Molecular balance experiment	a few percent	Estermann, Simpson and Stern[15]
K	Molecular balance experiment	a few percent	Estermann, Simpson and Stern[15]
n	Neutron fall experiment	a few percent	Dabbs, Harvey, Paya and Horstmann[16]
n	Storage ring balance	4%	Paul, Anton, Paul, Paul and Mampe[17]
Dy	Beam balance experiment	10^{-3}	Ni, Chou, Pan, Lin, Hwong, Ko, and Li[12]
Dy	Torsion balance experiment	1.4×10^{-2}	Chou, Ni and Wang[13]

Further, the experimental techniques needed to mount such an experiment are well established.

For the polarized body the rare earth magnetic alloys seem to be suitable. For example, the magnetic field at the nucleus of a Dy atom is about 5.5 M Gauss (in all the rare earth alloys, this field is typically close to the free 3+ ion value), and for ^{161}Dy (I=5/2) the first excited hyperfine level is at an energy E above the lowest level where $E/k = 37$ mK. Temperatures of the order of 10 mK are easily obtained using commercially available ^{3}He/^{4}He dilution refrigerators. Thus at this temperature, the population of the first excited hyperfine level is about $e^{-3.7}$, or 2.5×10^{-2}, of that of the lowest level, corresponding to more than 97% nuclear polarization of this odd isotope ^{161}Dy.

We propose and are working on a Dicke type equivalence principle experiment mounted inside an enclosure cooled by a dilution refrigerator with the suspended masses being nuclear polarized bodies cooled to 10 mK (see Fig. 1). The dilution refrigerator is to be a CryoVac model DRS-523-SS with a cooling power of 2 μW at 10 mK, 12 μW at 20 mK, and 20 μW at 25 mK. A can attached to and cooled by the mixing chamber, with a diameter of 8 cm and a length of 75 cm, should provide ample space to accommodate this experimental arrangement.

We will use an "optical lever" for the readout of the position of the suspended weights. A well collimated beam from a He-Ne laser will pass down the central access tube of the refrigerator and will return by the same path. Narrow pass interference filters at 77 K and 4.2 K will act as radiation shields. The power in the beam can be of the order of a few microwatts, this is within the sensitivity range of the charge coupled detectors used to monitor the returning beam position. A dielectric mirror on the suspended weights will reflect more than 99% of the

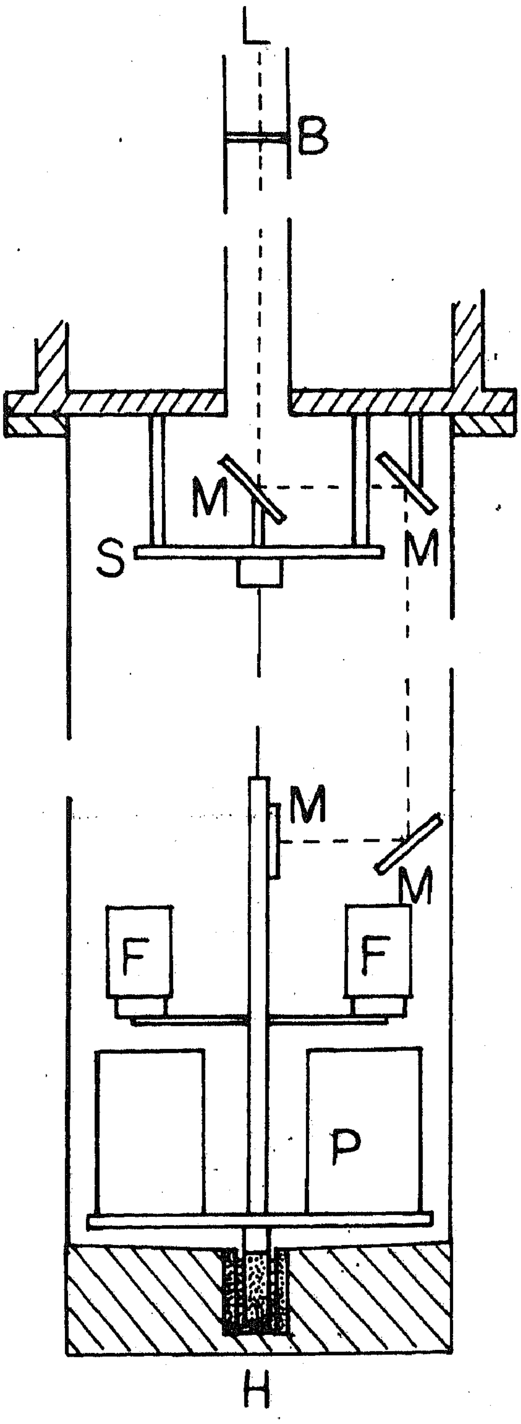

Fig. 1. Schematic diagram (not to scale) of a Dicke type experiment with nuclear polarized bodies. The experiment will be mounted in an enclosure fastened underneath and cooled by the mixing chamber of a dilution refrigerator. S is a radiation shield and mounting plate. P is a polarized mass. F are the electrodes for an electrostatic feedback system. H is the heat exchanger, of sintered silver immersed in liquid ^{3}He, for cooling the suspended assembly. The readout is by an optical lever, using a laser beam L and mirrors M. B is a narrow band optical filter and radiation shield. The lower fixed mirror and the fixed electrodes for the feedback are hung from the top mounting plate; the mounting is not shown. On the pan of the suspension there will be three masses with one polarized or four masses with two polarized. One polarized and one unpolarized are shown in the figure. To adapt this setup for measuring the force or torque between two polarized bodies we can put two polarized mass on the pan and two fixed polarized masses beside or underneath; the polarization of the fixed masses needs to be reversed from time to time.

incident power. Since the period of torsional oscillations is expected to be of the order of 2–5 minutes, the orientation of the suspended system need be sampled at intervals of the order of only 0.1 sec. reducing the heat input due to the laser beam to the order of nanowatts. An electrostatic servo system will be used to hold the suspended system at zero deflection. This will reduce some possible systematic errors and the torque will be obtained from the feedback voltage.

Feasibility studies indicate that the thermal radiation from 4.2 K should be of the order of 2 nW, and most of this will be intercepted by a radiation shield (in Fig. 1). Thermal contact between the suspended masses and the mixing chamber is to be by conduction through a pool of liquid ^{3}He in the bottom of the can. The bottleneck here is the Kapitza resistance between the liquid ^{3}He and the copper rod, attached to the bottom of the suspended assembly, which dips into this pool of liquid ^{3}He. The greatest temperature drop occurs at the interface between the liquid and this copper rod. A few grams of sintered silver attached to the bottom of this rod would have an area of 6 m^2, and would conduct 800 nW for a temperature drop of 1 mK.[19]

The temperature of the suspended masses can be measured by a nuclear orientation thermometer. The first choice for this is ^{54}Mn in nickel, which has a maximum sensitivity around 10 mK.[20] A strength of 0.4 μCi would generate about 1 nW of radioactive heating, and would enable the count rate to be determined to 1% in about 15 mins. This is quite adequate since the nuclear polarization varies very little with temperature around 10 mK, and the experiment lasts for a long time. A disadvantage of nickel is that it must be magnetized; this can be done with a small permanent magnet, or with one of the magnets used to polarize the rare earth alloy. Another nuclear orientation thermometer which does not require a magnet is ^{60}Co in a hexagonal cobalt single crystal, which is the second choice.

A substance which can achieve considerable nuclear polarization through cooling must necessarily have a large specific heat in this temperature range due to the hyperfine splitting. A representative calculation, made for 100 gm of Dy_6Fe_{23}, shows that typically to cool this specimen from 20 mK to 10 mK requires the removal of 3.6 mJ of heat. Most of this heat will come out at temperatures somewhat higher than 10 mK, where heat transfer is much more rapid (the Kapitza boundary resistance varies as $1/T^3$). If we assume that both ^{161}Dy and ^{163}Dy have a first excited hyperfine level at E/k=37 mK (for ^{163}Dy, E/k=51 mK), this gives an upper bound to the final cooling time, and also a simpler differential equation to solve. This equation is

$$\frac{dT}{dt} = -C(T^4 - T_0^4)\frac{1}{mR}\left(\frac{kT}{E}\right)^2 (1+e^{-E/kT})^2/e^{-E/kT}\,, \tag{3.1}$$

where T is the temperature of the suspended masses, T_0 is the final temperature (10 mK), m is the number of mols. to be cooled, and R is the gas constant. The parameter $C = A/4R_K T^3$, where A is the area of sinter on each side and R_K is the Kapitza boundary resistance. Since there are large uncertainties in the value of R_K, a more detailed calculation of cooling time is not warranted.

This equation was solved numerically, giving a cooling time from 20 mK to 12 mK of 1.5 hr, and 11 mK of 2.6 hr. Times for the final cooling down, even of the order of hours, are not out of line with the time scale of the experiment.

Dy_6Fe_{23} is a representative material for nuclear polarization. It is chosen because it is a compensated permanent ferrimagnet; the dysprosium lattice is magnetized to saturation while the bulk magnetic moment is close to zero. Thus, the Dy_6Fe_{23} is easily shielded with 3 layers of pure iron and μ-metal, or some similar material. The earth's magnetic field (including the laboratory field) can be shielded by μ-metal layers. Magnetic interaction with the world outside the 10 mK enclosure or with images, is thus minimized. Most of these magnetic interactions contribute a constant background to the torque, which can be subtracted, since the mode of operation is to be one where the suspended masses are kept at a fixed orientation by a servo mechanism.

Another substance which will be used is a holmium alloy. $HoFe_3$ and Ho_6Fe_{23} are also ferrimagnetic. ^{165}Ho has a very large hyperfine field and typically the peak in the Schottky specific heat occurs at a much higher temperature of the order of 10^2 mK. As a result, the nuclear specific heat in the 10–20 mK range is negligible and cooling to 10 mK is very rapid, of the order of minutes or less.

4. Discussions and Perspectives

For our millikelvin equivalence principle experiments, we plan first to use holmium iron compounds as polarized bodies, and afterwards Dy_6Fe_{23}, $PrCu_5$ and other rare-earth materials. An improvement of three orders or more of the equivalence principle test for polarized nucleus is expected.

Polarized beam,[22] polarized atomic fountains and traps can test the equivalence principle for polarized nuclei in the earth's gravitational field. It is desirable to pursue these methods since they can test the equivalence principle for free polarized nuclei.

Light nuclei have more polarized nucleons per nucleus. Possible methods using these species are worth investigating.

To measure the force or torque between two polarized bodies we can put two fixed polarized masses besides the two suspended polarized masses. For comparison we need to reverse the polarization of the fixed masses. $PrCu_5$ is suitable for this purpose, since the field needed is only a couple of hundred gauss, and the reversal can be achieved by reversing the current of a superconducting magnet. Dysprosium iron compounds and holmium iron compounds are not suitable for this purpose, since they need a couple of teslas for reversal.

For the detection of exotic long range spin-spin couplings, polarized ^{3}He is potentially an extremely sensitive detector. It is well known that ^{3}He, as a gas at room temperature, can be polarized by optical pumping, and that polarizations of the order of 60% are readily attained.[23] Furthermore, the relaxation time in aluminosilicate glass containers is very long — typically two or three days.[24] This long relaxation time allows the small effects of a weak interaction to build up cumulatively to a large value.

For example, an exotic spin-spin interaction would have the same effect as a magnetic field, and resonance, exactly like nuclear magnetic resonance, might be observed. We would therefore set up a nuclear magnetic resonance experiment using the exotic field in place of the oscillating magnetic field. We can take as a measure of the sensitivity the rotating field (H_1 in the standard notation of magnetic resonance) which will turn the magnetic moment, or polarization vector of the ^{3}He, through 90° in the same manner as it is turned through 90° by $\pi/2$ pulse. In ordinary nuclear magnetic resonance, this field, H_1, would be 1.78×10^{-9} Gauss (1.78×10^{-13} T). Thus, it would be possible to detect an exotic spin-spin coupling whose strength is equivalent to a magnetic field of this intensity.

There are two ways of observing this effect, either by the voltage induced in a pickup coil by the rotating ^{3}He nuclear spins, the usual method of nuclear magnetic resonance, or by measuring the component of the magnetic moment of ^{3}He along the direction of the static field (H_0 in the standard notation of magnetic resonance) using a squid. Assuming a sample of ^{3}He at 1 Torr in a glass bulb 3 cm diameter and a 10 turn pickup coil for a squid, the signal obtained when the polarization of the ^{3}He is rotated through 90° is about 100 ϕ_0. This is small, but it is independent of frequency. To detect the rotating magnetic moment of the ^{3}He by magnetic pickup is not easy, but not impossible using a lock-in amplifier with a long time constant. At low audio frequencies, a time constant of the order of 1000 s might seem desirable, while at much higher frequencies the rotating magnetic moment would be much easier to detect. There are, however, disadvantages to using much higher frequencies which we will now consider.

To produce a rotating exotic field, it would be necessary to rotate the polarization of a polarized body placed conveniently close to the sample of polarized ^{3}He. If such a body is a compensated ferrimagnet like Dy_6Fe_{23}, this might be done by rotating the whole body, but to achieve rates of rotation greater than about 100 Hz would be a formidable task, and at such low frequencies magnetic detection of the rotating ^{3}He magnetic moment would not be easy. Alternatively, it might be easier to rotate the polarization of a polarized body, without mechanically rotating the body itself, if this polarization were produced by some dynamic means. Such arrangements are complicated by the need to shield the ^{3}He from stray magnetic fields at least to a level of 10^{-9} Gauss.

An alternative scheme is to leave the polarized body, which is the source of the exotic field, in a fixed position, and perform the resonance in a rotating frame. In this arrangement, the polarized ^{3}He is in a steady field H_0 and a field H_1 rotating at an angular velocity ω in a plane perpendicular to H_0. When we transform to a coordinate system rotating with H_1, the effective field perpendicular to the plane of rotation is $H_0 - \omega/\gamma$, where γ is the gyromagnetic ratio of ^{3}He. In this coordinate system, the ^{3}He spins precess about a resultant field H_R, where $H_R^2 = H_1^2 + (H_0 - \omega/\gamma)^2$, which makes an angle θ with H_0 where $\tan\theta = H_1/(H_0 - \omega\gamma)$. If H_0, H_1, and ω now satisfy $H_1^2 = H_0(2\omega/\gamma - H_0)$, in the rotating coordinate system the ^{3}He spins precess about H_R with an angular velocity ω.

A static exotic field, equivalent to a magnetic field H_2, can be resolved into three components in the rotating system — a field $H_2 \sin\theta$ oscillating along the direction of H_R, and two fields, of magnitudes 1/2 $H_2(1\pm\cos\theta)$ rotating in opposite directions about H_R. One of the rotating component will induce resonance in the usual way, and the other two will have negligible effect. In the laboratory system, the magnetic moment of the ^{3}He will be seen to pulsate along the direction of H_0, and the nuclear exotic resonance will manifest itself as a change in the amplitude of this pulsation.

Such an arrangement would enable the resonance to be carried out at conveniently higher frequencies. There would still be, however, formidable problems of magnetic shielding, and problems of establishing the accuracy of the fields H_0 and H_1 and keeping them stable and uniform. Nevertheless, so many ingenious variations on the theme of magnetic resonance have been devised over the years, that it is very probable that some other experimental arrangement might well be more conveniently adapted to nuclear exotic resonance.

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